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Canonical Global Symmetry and Quantal Conservation Laws for a System with Singular Higher Order Lagrangian

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Starting from the phase-space generating functional of the Green function for a system with singular higher order Lagrangian, the generalized canonical Ward identities under the global symmetry transformation in phase space is deduced. The local transformation connected with this global symmetry transformation is studied, and the quantal conservation laws are obtained for such a system. We give a preliminary application to higher derivative Yang–Mills theory; a generalized quantal BRS conserved quantity is found.

1. INTRODUCTION

The connection between continuous global symmetry and conservation laws is usually referred to as the Noether theorem in classical theories. Ward identities (or Ward–Takahashi identities) play an important role in modern quantum field theories (Ward, 1950; Takahashi, 1957; Slavnov, 1972; Taylor, 1971). These identities have been generalized to supersymmetry (Joglekhar, 1991) and superstring theories (Danilov, 1991) and other problems. All of these discussions for the Noether theorem and Ward identities in the functional integration method (Surra and Young, 1973; Young, 1987; Lhallabi, 1989) are usually based on the examination of the Lagrangian in configuration space and the corresponding transformation expressed in terms of Lagrange's variables. The generalization of the Noether theorem to a system with a singular Lagrangian in terms of canonical variables was given in Li (1993). Phase-space path integrals are more basic than configuration-space path integrals (Mizrahi, 1978). While the phase-space generating functional cannot

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be simplified by carrying out explicit integration over the canonical momenta, even if the integration over the momenta can be carried out, the effective Lagrangian sometimes shows a singularity with a δ-function (Lee and Yang, 1962; Gerstein *et al.*, 1971; Du *et al.*, 1980). Thus, the investigation of the symmetry properties of the system in the phase space for the quantum theories has more fundamental sense. Canonical local symmetry in a system and canonical Ward identities were discussed in Li (1994, 1995). Dynamical systems described in terms of a higher order Lagrangian obtained by many authors are of much interest in connection with gauge theories, gravity, supersymmetry, string models, and other problems (Li, 1993). Here, canonical global symmetry for a system with a singular higher order Lagrangian will be further investigated. Based on the phase-space generating functional of the Green function for a system with singular higher order Lagrangian, the generalized canonical Ward identities under the global symmetry transformation in phase space are deduced. The realization of a canonical Noether theorem at the quantum level for such systems is given. Applying our formulation to higher derivative Yang–Mills theory, we obtain a generalized quantal BRS conserved quantity.

2. CANONICAL WARD IDENTITIES FOR GLOBAL SYMMETRY

Let us consider a dynamical system described by a singular Lagrangian

$$\mathscr{L}(\phi^\alpha, \phi^\alpha_{,\mu}, \cdots \phi^\alpha_{,\mu(N)}), \qquad \phi^\alpha_{,\mu(m)} = \underbrace{\partial_\mu \cdots \partial_\sigma}_{m} \phi^\alpha$$

Due to the singularity of the Lagrangian, the motion of this system is restricted to a hypersurface of the phase space, determined by a set of constraints. The generating functional of the Green's function for this system can be written as (Li, 1994; Gitman and Tyutin, 1990)

$$Z[J, K] = \int \mathscr{D}\phi^\alpha_{(s)}\, \mathscr{D}\pi^{(s)}_\alpha\, \mathscr{D}\lambda_m\, \mathscr{D}C_l\, \mathscr{D}\overline{C}_k \times \exp\left[i \int d^4x\, (\mathscr{L}^p_{\text{eff}} + J^{(s)}_\alpha \phi^\alpha_{(s)} + K^{(s)}_\alpha \pi^\alpha_{(s)})\right] \tag{1}$$

where I^p_{eff} is an effective canonical action

$$I^p_{\text{eff}} = \int d^4x\, \mathscr{L}^p_{\text{eff}} = \int d^4x \left[\pi^{(s)}_\alpha \dot{\phi}^\alpha_{(s)} - \mathscr{H}_c + \lambda_m \Phi_m + \frac{1}{2}\int d^4y\, \overline{C}_k(x)\{\Phi_k(x), \Phi_l(y)\} C_l(y)\right] \tag{2}$$

and $\pi_\alpha^{(s)}$ is the canonical momentum determined by the Ostrogradsky transformation, $\mathcal{H}_c$ is the canonical Hamiltonian density, $\Phi = \{\Phi_m\}$ is a set of all constraints (for a theory with second-class constraints) or the set of constraints and gauge conditions (for a theory with first-class constraints), $\{\cdot, \cdot\}$ denotes generalized Poisson bracket, $C_l(x)$ and $\overline{C}_k(x)$ are Grassmann variables, and $\lambda_m(x)$ are Lagrange multipliers. For the sake of simplicity, we put

$$\varphi_{(s)}^\alpha = (\phi_{(s)}^\alpha, \lambda_m, C_l, C_k) \qquad \text{and} \qquad j_\alpha^{(s)} = (J_\alpha^{(s)}, U_m, \xi_k, \bar{\xi}_l)$$

where U_m, ξ_k, and $\bar{\xi}_l$ are exterior sources with respect to λ_m, $\overline{C}_k$, and C_l, respectively; then the expression (1) can be written as

$$Z[j, K] = \int \mathcal{D}\varphi_{(s)}^\alpha\, \mathcal{D}\pi_\alpha^{(s)} \exp\left[i \int d^4x\, (\mathcal{L}_{\text{eff}}^p + j_\alpha^{(s)}\varphi_{(s)}^\alpha + K_{(s)}^\alpha \pi_\alpha^{(s)})\right] \tag{3}$$

Let us consider a global symmetry transformation in extended phase space whose infinitesimal transformation is given by

$$\begin{cases} x^{\mu'} = x^\mu + \Delta x^\mu = x^\mu + \epsilon_\sigma \tau^{\mu\sigma}(x, \varphi_{(s)}^\alpha, \pi_\alpha^{(s)}) \\ \varphi_{(s)}^{\alpha'}(x') = \varphi_{(s)}^\alpha(x) + \Delta\varphi_{(s)}^\alpha(x) = \varphi_{(s)}^\alpha(x) + \epsilon_\sigma \xi_{(s)}^{\alpha\sigma}(x, \varphi_{(s)}^\alpha, \pi_\alpha^{(s)}) \\ \pi_\alpha^{(s)'}(x') = \pi_\alpha^{(s)}(x) + \Delta\pi_\alpha^{(s)}(x) = \pi_\alpha^{(s)}(x) + \epsilon_\sigma \eta_\alpha^{(s)\sigma}(x, \varphi_{(s)}^\alpha, \pi_\alpha^{(s)}) \end{cases} \tag{4}$$

where ϵ_σ ($\sigma = 1, 2, \ldots, r$) are infinitesimal arbitrary parameters, and $\tau^{\mu\sigma}$, $\xi_{(s)}^{\alpha\sigma}$, and $\eta_\alpha^{(s)\sigma}$ are functions of x, $\varphi_{(s)}^\alpha$, and $\pi_\alpha^{(s)}$. It is supposed that the effective canonical action is invariant under the transformation (4) and the Jacobian of the transformation (4) is equal to unity. The generating functional (3) is invariant under the transformation (4); thus, one gets

$$\begin{aligned} Z[j, K] = \int \mathcal{D}\varphi_{(s)}^\alpha\, \mathcal{D}\pi_\alpha^{(s)} \Bigg(1 + i\epsilon_\sigma \int d^4x \Bigg\{ & j_\alpha^{(s)}\left(\xi_{(s)}^{\alpha\sigma} - \tau^{\mu\sigma}\partial_\mu \frac{\delta}{\delta j_\alpha^{(s)}}\right) \\ & + K_{(s)}^\alpha\left(\eta_\alpha^{(s)\sigma} - \tau^{\mu\sigma}\partial_\mu \frac{\delta}{\delta K_{(s)}^\alpha}\right) \\ & + \partial_\mu\left[\tau^{\mu\sigma}\left(j_\alpha^{(s)} \frac{\delta}{\delta j_\alpha^{(s)}} + K_{(s)}^\alpha \frac{\delta}{\delta K_{(s)}^\alpha}\right)\right]\Bigg\}\Bigg)_{\varphi_{(s)}^\alpha \to -i\delta/\delta j_\alpha^{(s)},\, \pi_\alpha^{(s)} \to -i\delta/\delta K_{(s)}^\alpha} Z[j, K] \end{aligned} \tag{5}$$

Consequently, the generating functional satisfies the following generalized canonical Ward identities:

$$\int d^4x \left\{ j_\alpha^{(s)}\left(\xi_{(s)}^{\alpha\sigma} - \tau^{\mu\sigma}\partial_\mu \frac{\delta}{\delta j_\alpha^{(s)}}\right) + K_{(s)}^\alpha\left(\eta_\alpha^{(s)\sigma} - \tau^{\mu\sigma}\partial_\mu \frac{\delta}{\delta K_{(s)}^\alpha}\right) + \partial_\mu\left[\tau^{\mu\sigma}\left(j_\alpha^{(s)} \frac{\delta}{\delta j_\alpha^{(s)}} + K_{(s)}^\alpha \frac{\delta}{\delta K_{(s)}^\alpha}\right)\right]\right\}_{\varphi_{(s)}^\alpha \to -i\delta/\delta j_\alpha^{(s)},\, \pi_\alpha^{(s)} \to -i\delta/\delta K_{(s)}^\alpha} Z[j, K] = 0 \tag{6}$$

Functionally differentiating (6) with respect to the exterior sources $j_\alpha^{(0)}$ many times and setting all exterior sources equal to zero, one can obtain some relationships among the Green functions.

3. CONSERVATION LAWS AT THE QUANTUM LEVEL

It is supported that the effective canonical action is invariant under the global transformation (4). Let us consider the corresponding local transformation:

$$\begin{cases} x^{\mu'} = x^\mu + \Delta x^\mu = x^\mu + \epsilon_\sigma(x)\tau^{\mu\sigma}(x, \varphi_{(s)}^\alpha, \pi_\alpha^{(s)}) \\ \varphi_{(s)}^{\alpha'}(x') = \varphi_{(s)}^\alpha(x) + \Delta\varphi_{(s)}^\alpha(x) = \varphi_{(s)}^\alpha(x) + \epsilon_\sigma(x)\xi_{(s)}^{\alpha\sigma}(x, \varphi_{(s)}^\alpha, \pi_\alpha^{(s)}) \\ \pi_\alpha^{(s)'}(x') = \pi_\alpha^{(s)}(x) + \Delta\pi_\alpha^{(s)}(x) = \pi_\alpha^{(s)}(x) + \epsilon_\sigma(x)\eta_\alpha^{(s)\sigma}(x, \varphi_{(s)}^\alpha, \pi_\alpha^{(s)}) \end{cases} \tag{7}$$

where $\epsilon_\sigma(x)$ ($\sigma = 1, 2, \ldots, r$) are infinitesimal arbitrary functions; the values of $\epsilon_\sigma(x)$ and their derivatives up to required order on the boundary of time-space domain vanish. Under the transformation (7) the variation of the effective canonical action is given by Li (1993)

$$\begin{aligned} \delta I_{\text{eff}}^P = \int d^4x\, \epsilon_\sigma(x)\{&(-\dot{\pi}_\alpha^{(s)} - \delta H_{\text{eff}}/\delta\varphi_{(s)}^\alpha)(\xi_{(s)}^{\alpha\sigma} - \varphi_{(s),\mu}^\alpha\tau^{\mu\sigma}) \\ &+ (\dot{\varphi}_{(s)}^\alpha - \delta H_{\text{eff}}/\delta\pi_\alpha^{(s)})(\eta_\alpha^{(s)\sigma} - \pi_{\alpha,\mu}^{(s)}\tau^{\mu\sigma}) \\ &+ \partial_\mu[(\pi_\alpha^{(s)}\dot{\varphi}_{(s)}^\alpha - \mathcal{H}_{\text{eff}})\tau^{\mu\sigma}] + D[\pi_\alpha^{(s)}(\xi_{(s)}^{\alpha\sigma} - \varphi_{(s),\mu}^\alpha\tau^{\mu\sigma})]\} \\ &+ \int d^4x\, \{[(\pi_\alpha^{(s)}\dot{\varphi}_{(s)}^\alpha - \mathcal{H}_{\text{eff}})\tau^{\mu\sigma}]\partial_\mu\epsilon_\sigma(x) \\ &+ \pi_\alpha^{(s)}(\xi_{(s)}^{\alpha\sigma} - \varphi_{(s),\mu}^\alpha\tau^{\mu\sigma})D\epsilon_\sigma(x) \end{aligned} \tag{8}$$

where $D = d/dt$, and H_{eff} is the Hamiltonian connected with $L_{\text{eff}}^P = \int d^3x\, \mathcal{L}_{\text{eff}}^P$. Since we have assumed that the effective canonical action is invariant under the global transformation (4), then the first integral in expression (8) is equal to zero. The Jacobian of the transformation (7) is denoted by $J[\varphi, \pi, \epsilon]$; the invariance of the generating functional (3) under the transformation

(7) implies that $\delta Z/\delta\epsilon_\sigma(x) = 0$. We perform the integration by parts of the right-hand side of the remaining terms in expression (8), after which we substitute the result of δI^p_{eff} into (3) and functionally differentiate the obtained result with respect to $\epsilon_\sigma(x)$; we have

$$\int \mathcal{D}\varphi^\alpha_{(s)}\, \mathcal{D}\pi^{(s)}_\alpha \,\{\partial_\mu[(\pi^{(s)}_\alpha\dot{\varphi}^\alpha_{(s)} - \mathcal{H}_{\text{eff}})\tau^{\mu\sigma}] + D[\pi^{(s)}_\alpha(\xi^{\alpha\sigma}_{(s)} - \varphi^\alpha_{(s),\mu}\tau^{\mu\sigma})] - J^\sigma_0 - M^\sigma\}\exp\left[i\int d^4x\,(\mathcal{L}^p_{\text{eff}} + j^{(s)}_\alpha\varphi^\alpha_{(s)} + K^\alpha_{(s)}\pi^{(s)}_\alpha)\right] = 0 \tag{9}$$

where

$$J^\sigma_0 = -i\delta J[\varphi, \pi, \epsilon]/\delta\epsilon_\sigma(x)\,|_{\epsilon_\sigma(x)=0} \tag{10}$$

$$M^\sigma = j^{(s)}_\alpha(\xi^{\alpha\sigma}_{(s)} - \varphi^\alpha_{(s),\mu}\tau^{\mu\sigma}) + K^\alpha_{(s)}(\eta^{(s)\sigma}_\alpha - \pi^{(s)}_{\alpha,\mu}\tau^{\mu\sigma}) \tag{11}$$

Functionally differentiating (9) with respect to $j^{(0)}_\alpha$ n times, one gets

$$\int \mathcal{D}\varphi^\alpha_{(s)}\, \mathcal{D}\pi^{(s)}_\alpha \Big(\{\partial_\mu[(\pi^{(s)}_\alpha\dot{\varphi}^\alpha_{(s)} - \mathcal{H}_{\text{eff}})\tau^{\mu\sigma}] + D[\pi^{(s)}_\alpha(\xi^{\alpha\sigma}_{(s)} - \varphi^\alpha_{(s),\mu}\tau^{\mu\sigma})] - J^\sigma_0 - M^\sigma\}\varphi^\alpha(x_1)\varphi^\alpha(x_2)\cdots\varphi^\alpha(x_n) - i\sum_j \varphi^\alpha(x_1)\varphi^\alpha(x_2)\cdots\varphi^\alpha(x_{j-1})\varphi^\alpha(x_{j+1})\cdots\varphi^\alpha(x_n) \times N^{\alpha\sigma}\delta(x - x_j)\Big)\exp\left[i\int d^4x\,(\mathcal{L}^p_{\text{eff}} + j^{(s)}_\alpha\varphi^\alpha_{(s)} + K^\alpha_{(s)}\pi^{(s)}_\alpha)\right] = 0 \tag{12}$$

where

$$N^{\alpha\sigma} = \xi^{\alpha\sigma}_{(0)} - \varphi^\alpha_{,\mu}\tau^{\mu\sigma} \tag{13}$$

Let us set all exterior sources equal to zero in expression (12); $J^{(s)}_\alpha = K^\alpha_{(s)} = 0$; we obtain

$$\langle 0|T^*\{\partial_\mu[(\pi^{(s)}_\alpha\dot{\varphi}^\alpha_{(s)} - \mathcal{H}_{\text{eff}})\tau^{\mu\sigma}] + D[\pi^{(s)}_\alpha(\xi^{\alpha\sigma}_{(s)} - \varphi^\alpha_{(s),\mu}\tau^{\mu\sigma})] - J^\sigma_0\}\varphi^\alpha(x_1)\varphi^\alpha(x_2)\cdots\varphi^\alpha(x_n)|0\rangle = i\sum_j \langle 0|T^*[\varphi^\alpha(x_1)\varphi^\alpha(x_2)\cdots\varphi(x_{j-1})\varphi(x_{j+1})\cdots\varphi(x_n)N^{\alpha\sigma}|0\rangle\delta(x - x_j) \tag{14}$$

where the symbol T^* stands for the convariantized T product (Surra and Young, 1973; Young, 1987). Fixing t and letting

$$t_1, t_2, \ldots, t_m \to +\infty, \qquad t_{m+1}, t_{m+2}, \ldots, t_n \to -\infty$$

and using the reduction formula (Surra and Young, 1973; Young, 1987), we can write expression (14) as

$$\langle \text{out}, m | \{ \partial_\mu [(\pi_\alpha^{(s)} \dot{\varphi}_{(s)}^\alpha - \mathcal{H}_{\text{eff}}) \tau^{\mu\sigma}] + D[\pi_\alpha^{(s)} (\xi_{(s)}^{\alpha\sigma} - \varphi_{(s),\mu}^\alpha \tau^{\mu\sigma})] - J_0^\sigma \} | n - m, \text{in} \rangle = 0 \tag{15}$$

Since m and n are arbitrary, this implies

$$\partial_\mu [(\pi_\alpha^{(s)} \dot{\varphi}_{(s)}^\alpha - \mathcal{H}_{\text{eff}}) \tau^{\mu\sigma}] + D[\pi_\alpha^{(s)} (\xi_{(s)}^{\alpha\sigma} - \varphi_{(s),\mu}^\alpha \tau^{\mu\sigma})] = J_0^\sigma \tag{16}$$

It is supposed that the Jacobian of transformation (7) is a constant [or independent of $\epsilon_\sigma(x)$]; in this case, $J_0^\sigma = 0$. We take the integral of the expression (16) on three-dimensional space. If we assume that the fields have a configuration which vanishes rapidly at spatial infinity, according to Gauss' theorem, we obtain the following conserved quantity at the quantum level:

$$Q^\sigma = \int d^3x \, [\pi_\alpha^{(s)} (\xi_{(s)}^{\alpha\sigma} - \varphi_{(s),k}^\alpha \tau^{k\sigma}) - \mathcal{H}_{\text{eff}} \tau^{0\sigma}] \tag{17}$$

This result holds for anomaly-free theories.

The conservation law (17) in the quantum case corresponds to the classical conservation laws deriving from the canonical Noether theorem (Li, 1993). In general, $\mathcal{H}_{\text{eff}}$ differs from the canonical Hamiltonian $\mathcal{H}_c$ and the Jacobian of the transformation (7) may not be a constant; then the conserved quantity (17) is different from the classical ones. The connection between the symmetries and conservation laws in classical theories in general is no longer preserved in quantum theories.

The advantage of the above formalism for obtaining conserved charges at the quantum level is that we do not need to carry out explicit integration over the canonical momenta in the phase-space generating functional. In the general case it is difficult or impossible to carry out these integrations.

4. AN APPLICATION

We consider Yang–Mills theory with higher order derivative whose Lagrangian is given by (Li, 1994; Gitman and Tyutin, 1990)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4\Lambda^2} D_{b\mu}^a F_{\nu\lambda}^b D_c^{a\mu} F^{c\lambda\nu} \tag{18}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f_{bc}^a A_\mu^b A_\nu^c \tag{19}$$

$$D_{b\mu}^a = \delta_b^a \partial_\mu + f_{cb}^a A_\mu^c \tag{20}$$

In the Coulomb gauge the phase-space generating functional of the Green functions for this system can be written as (Li, 1994)

$$Z[J, \xi, \bar{\xi}, \eta] = \int \mathfrak{D}A^a_\mu \, \mathfrak{D}A^a_{(1)\mu} \, \mathfrak{D}\pi^\mu_a \, \mathfrak{D}\pi^{(1)\mu}_a \, \mathfrak{D}C^a \, \mathfrak{D}\bar{C}^a \, \mathfrak{D}\lambda_m \, \delta(\Phi^G_{a1})\delta(\Phi^G_{a2}) \times \exp\left[i \int d^4x \, (\bar{\mathcal{L}}^p_{\text{eff}} + J^\mu_a A^a_\mu + \bar{C}^a \xi_a + \bar{C}^a \xi_a + \eta^m \lambda_m) \right] \quad (21)$$

where

$$\bar{\mathcal{L}}^p_{\text{eff}} = \mathcal{L}^p + \bar{\mathcal{L}}_m + \mathcal{L}_{gh} \quad (22)$$

$$\mathcal{L}^p = \pi^\mu_a \dot{A}^a_\mu + \pi^{(1)\mu}_a \dot{A}^a_{(1)\mu} - \mathcal{H}_c \quad (23)$$

$$\bar{\mathcal{L}}_m = \lambda^a_1 \Phi^{(1)}_{a1} + \lambda^a_2 \Phi^{(2)}_a \quad (24)$$

$$\mathcal{L}_{gh} = 2\bar{C}^a D^a_{bi} \partial_i C^b \quad (25)$$

$\mathcal{H}_c$ is the canonical Hamiltonian density for the Lagrangian (18), π^μ_a and $\pi^{(1)\mu}_a$ are the canonical momenta conjugate to A^a_μ and $A^a_{(1)\mu} = \dot{A}^a_\mu$, respectively, and $\{\Phi\}$ and $\{\Phi^G\}$ are constraints and gauge conditions, respectively (Gitman and Tyutin, 1990). In expression (21) we have introduced sources J^μ_a only to fields A^a_μ. The theory is independent of the choice of gauge conditions (Sundermeyer, 1982); the Φ^G_{ai} $(i = 1, 2)$ can be replaced by $\Phi^{G'}_{ai} = \Phi^G_{ai} - p_{ai}$, where $p_{ai}(x)$ are independent of the gauge. We consider instead of the measure in (21) one obtained by performing the Gaussian average

$$\int \mathfrak{D}p_{a1} \, \mathfrak{D}p_{a2} \exp\left[-i \int (p^2_{ai}/2\alpha_i) \, d^4x \right] \quad (26)$$

over the measure $\mathfrak{D}\mu$ defined by

$$\mathfrak{D}\mu = \mathfrak{D}A^a_\mu \, \mathfrak{D}A^a_{(1)\mu} \, \mathfrak{D}\pi^\mu_a \, \mathfrak{D}\pi^{(1)\mu}_a \, \mathfrak{D}C^a \, \mathfrak{D}\bar{C}^a \, \mathfrak{D}\lambda_m \times \delta(\Phi^G_{a1} - p_{a1})\delta(\Phi^G_{a2} - p_{a2}) \quad (27)$$

This amounts to the substitution of the generating functional (21) into the following expression:

$$Z[J, \xi, \bar{\xi}, \eta] = \int \mathfrak{D}A^a_\mu \, \mathfrak{D}A^a_{(1)\mu} \, \mathfrak{D}\pi^\mu_a \, \mathfrak{D}\pi^{(1)\mu}_a \, \mathfrak{D}C^a \, \mathfrak{D}\bar{C}^a \, \mathfrak{D}\lambda_m \times \exp\left[i \int d^4x \, (\mathcal{L}^p_{\text{eff}} + J^\mu_a A^a_\mu + \bar{C}^a \xi_a + \bar{\xi}_a C^a + \eta^m \lambda_m) \right] \quad (28)$$

where

$$\mathcal{L}^p_{\text{eff}} = \mathcal{L}^p + \mathcal{L}_f + \overline{\mathcal{L}}_m + \mathcal{L}_{gh} \tag{29}$$

$$\mathcal{L}_f = -\frac{1}{2\alpha_2}(\Phi^G_{a2})^2 = -\frac{1}{2\alpha_i}(\partial^i A^a_i)^2 \tag{30}$$

$$\overline{\mathcal{L}}_m = \lambda^a_1 \Phi^{(1)}_{a1} + \lambda^a_2 \Phi^{(2)}_{a2} - \frac{1}{2\alpha_1}(\Phi^G_{a1})^2 \tag{31}$$

Let us consider the BRS transformation in phase space:

$$\begin{cases} \delta A^a_\mu = D^a_{b\mu} C^b \tau, & \delta A^a_{(1)\mu} = \partial_0 (D^a_{b\mu} C^b \tau) \quad \text{(32a)} \\ \delta \pi^\mu_a = f^a_{be} \pi^\mu_e C^b \tau - f^a_{be} \pi^{(1)\mu}_e \dot{C}^b \tau, & \delta \pi^{(1)\mu}_a = f^a_{be} \pi^{(1)\mu}_e C^b \tau \quad \text{(32b)} \\ \delta C^a = \frac{1}{2} \tau f^a_{be} C^b C^e, & \delta \overline{C}^a = -(\tau/\alpha_2)\partial^i A^a_i \quad \text{(32c)} \end{cases}$$

where τ is a Grassmann parameter. Since $\Phi^{(1)}_{a1}$ and $\Phi^{(2)}_{a2}$ are first-class constraints, and the change of first-class constraints under the gauge transformation (32a) is within the constraint hypersurface (Li, 1995); thus, $\delta\overline{\mathcal{L}}_m \approx 0$ and $\delta\mathcal{L}_f \approx 0$ under the transformation (32). That is to say, the variation of the effective canonical action I^p_{eff} is $\delta I^p_{\text{eff}} \approx 0$ under the transformation (32), where the sign $\approx$ means equality on the constrained hypersurface (including gauge constraints). The Jacobian of the transformation (32) is equal to unity. According to (17), we obtain the generalized BRS conserved quantity at the quantum level

$$Q = \int d^3x \, [\pi^\mu_a \delta A^a_\mu + \pi^{(1)\mu}_a \delta A^a_{(1)\mu} + \pi_a \delta C^a + \overline{\pi}_a \delta \overline{C}^a] \tag{33}$$

where

$$\pi^0_a = \frac{1}{\Lambda^2} D^a_{bj} D^e_{b0} F^{j0}_e$$

$$\pi^i_a = \frac{1}{\Lambda^2}(D^{bj}_a D^e_{bj} F^{0i}_e + D^a_{bj} D^e_{b0} F^{ij}_e) - D^b_{a0} \pi^{(1)i}_b + F^{0i}_a \tag{34}$$

$$\pi^{(1)0}_a = 0, \qquad \pi^{(1)i}_a = \frac{1}{\Lambda^2} D^a_{bj} F^{ij}_b$$

$$\pi_a = -\dot{\overline{C}}^a, \qquad \overline{\pi}_a = D^a_{b0} C^b \tag{35}$$

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